The Hunt For Free-floating Black Holes

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Free-floating black holes

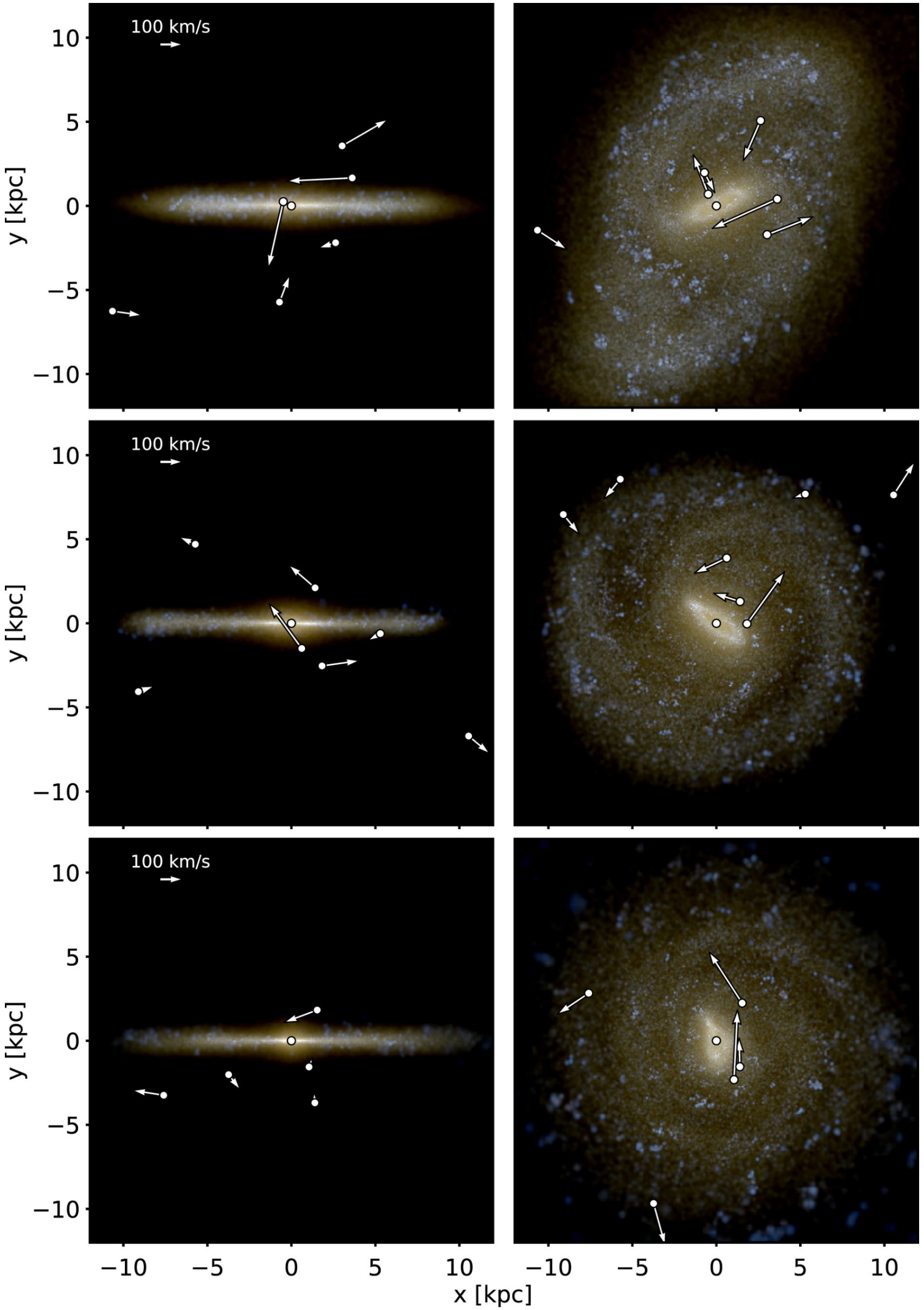
- **Free-floating/ Wandering black holes** - not sitting at the centres of galactic halos.

- 2 types-

  1. **Primordial black holes** (cosmological phase transition)
  2. **Formed by galaxy evolution**

  (ejection from binary BH merger, 3-body interaction of SMBH triplet, SMBHs from stripped satellite galaxies)

- Cosmological simulations e.g. **Romulus25 (Tremmel et al. 2018)** suggest ~ 10 wandering supermassive $(M \gtrsim 10^6 M_\odot)$ black holes in Milky Way sized galactic halos.



Milky Way sized halo in Romulus 25 cosmological simulation

Tremmel et al. 2018

- Dynamical friction weak → long inspiral time

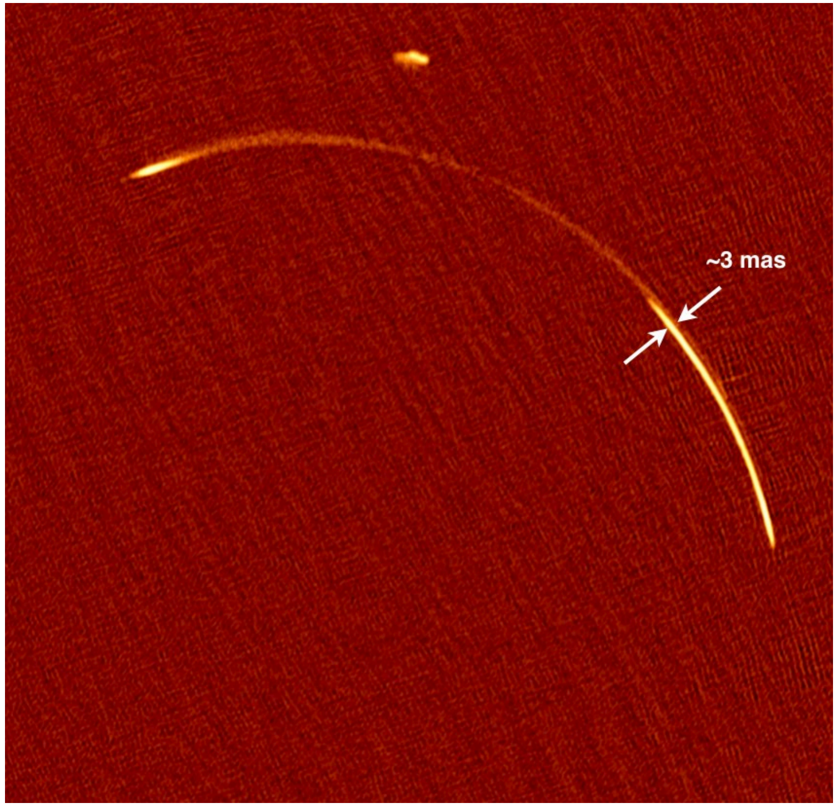
- Not enough matter → accretion disk/ tidal disruption events rare

- Observations of high velocity compact clouds **(Takekawa et al. 2018)** and X-ray observations of transient events like TDEs **(Lin et al. 2016)** can indicate their presence, but not enough for statistics.

How to **detect these black holes** and put reasonable **constraints** on their **number distribution**?

Gravitational lensing

Massive black holes can be detected by their distorting effects on the lensing arcs formed by dark matter halos.

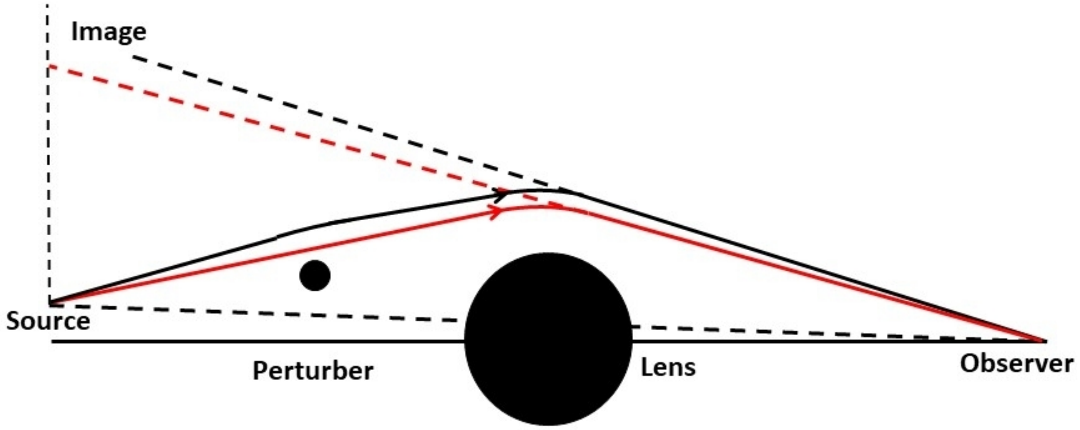


Observed
with VLBI

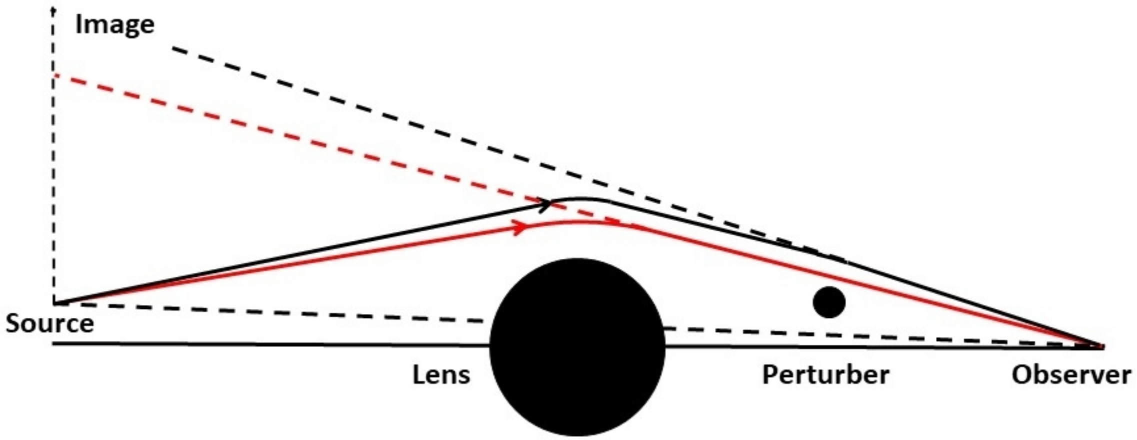
John McKean, on behalf of
SHARP collaboration

U. Banik et al. 2019

Double lens configuration

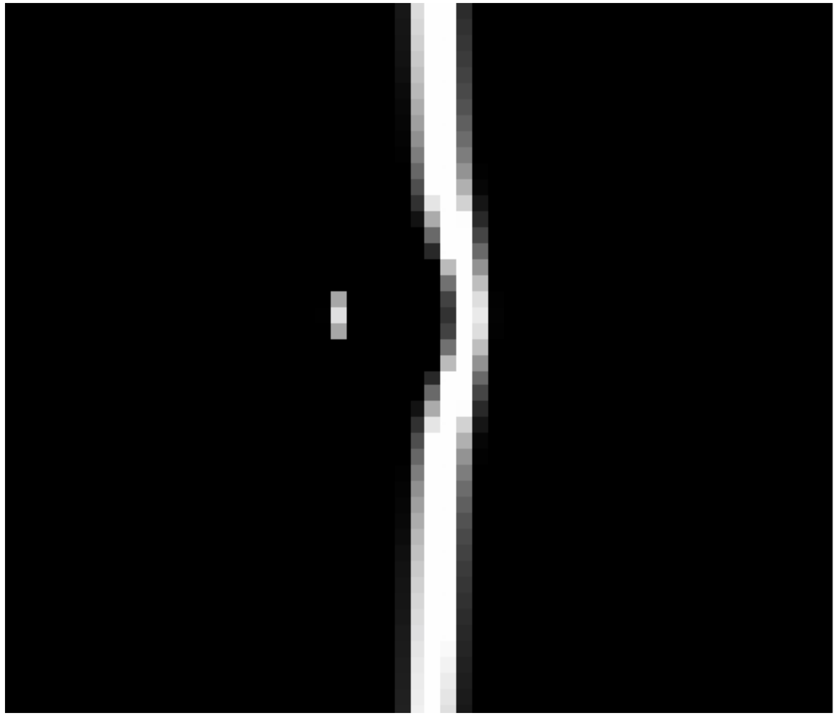


Background

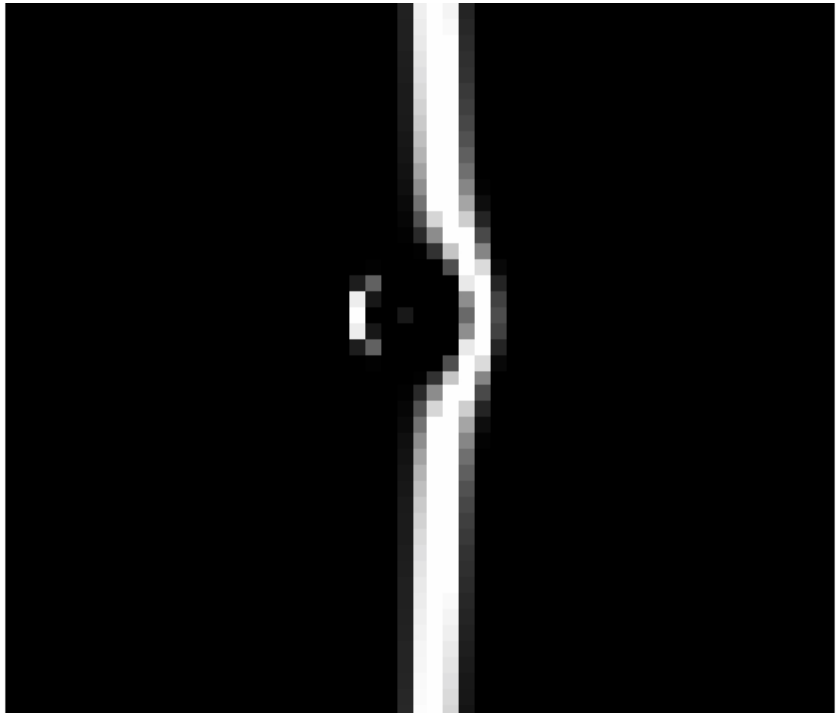


Foreground

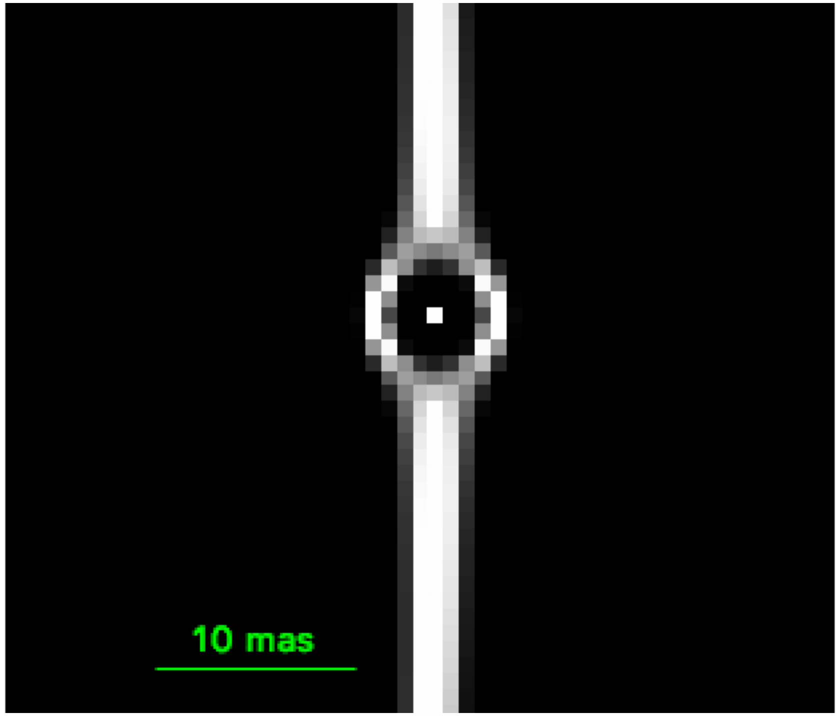
$\Delta\beta_{\mathrm{P}} = -3.2$ mas



$\Delta\beta_{\mathrm{P}} = -1.6$ mas



$\Delta\beta_{\mathrm{P}} = 0$ mas



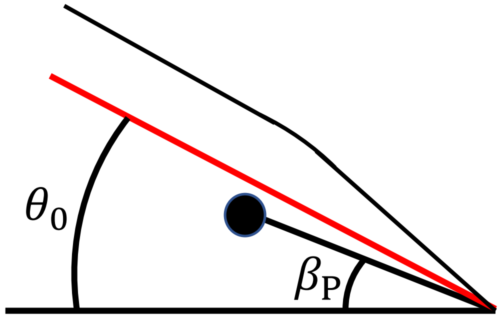
$10^6 M_\odot$ black hole

$M_{\mathrm{L}}(\theta_E) = 10^{12} M_\odot, z_{\mathrm{P}} = 0.5, z_{\mathrm{L}} = 0.881, z_{\mathrm{S}} = 2.056$

Boxcar filtering with
FWHM of 0.8 mas

GRAVLENS (Keeton 2001)

Angular impact parameter
$\Delta\beta_{\mathrm{P}} = \theta_0 - \beta_{\mathrm{P}}$



Machinery

- Take the **double lens equation** in presence and absence of the perturber.

- Lens models-

  1. **Primary lens**- **Dark matter halo (Isothermal sphere)**
  2. **Perturber**- **Schwarzschild black hole**

- Express the **perturbation** in the image angle as a function of the **angular impact parameter**, **redshift** and **mass of the black hole**.

- Obtain the condition for detecting the perturbation.

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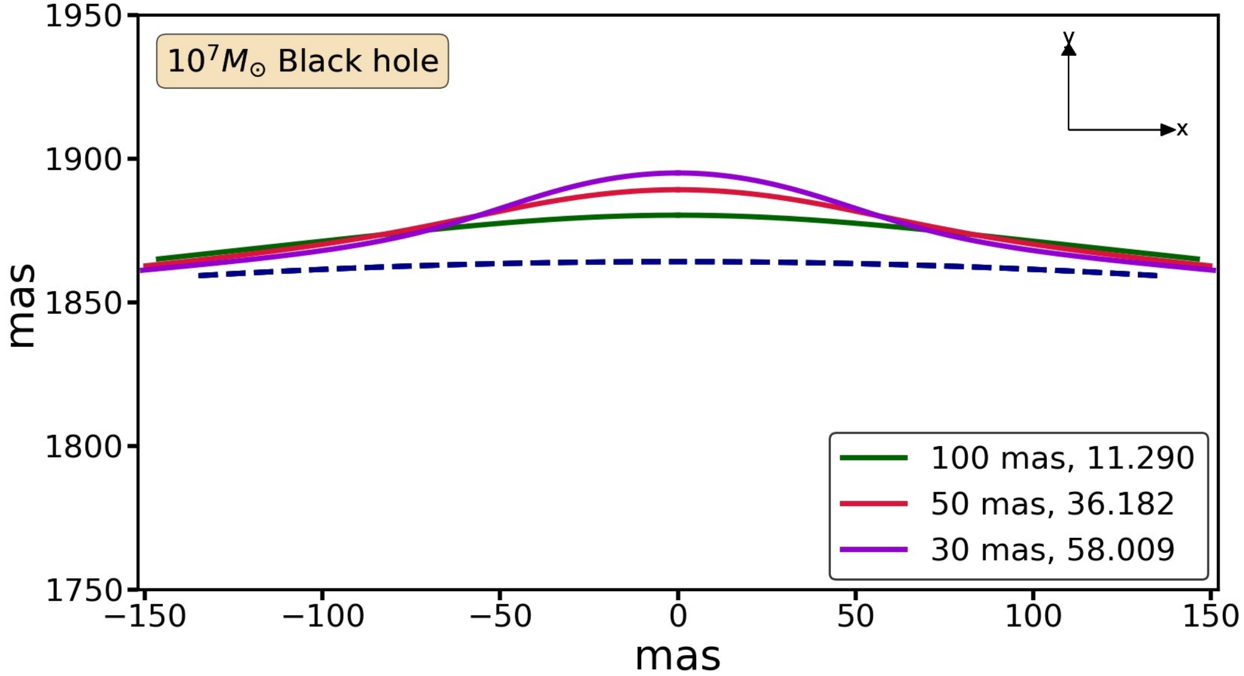
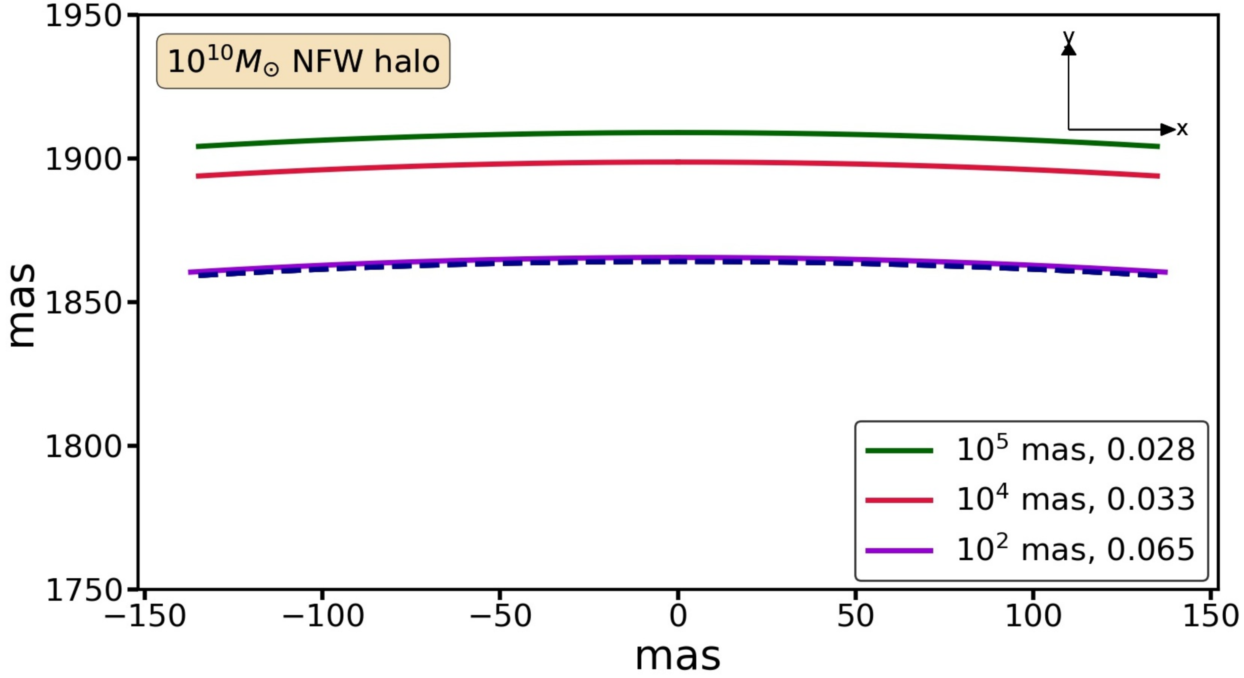


Black holes are compact.

Localized distortion of the arc



Dark matter subhalos are diffuse.

Overall displacement of the arc

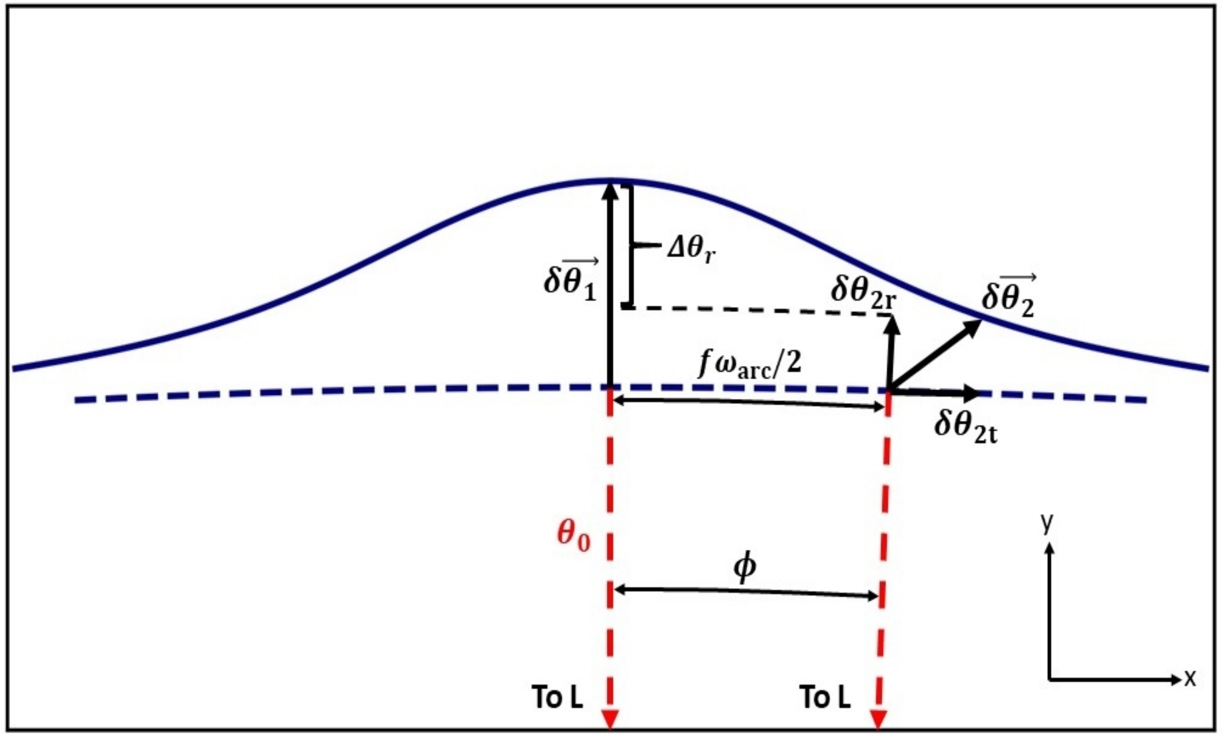
Detectability of the black hole

1. Perturbation needs to be resolved- **sub-milliarcsecond resolution** of VLBI required.

2. Localized perturbation

$$\boxed{\boldsymbol{\Delta\theta_{\mathrm{r}} = \delta\theta_r(\vec{\theta}_1) - \delta\theta_r(\vec{\theta}_2) \geq \mathcal{R}/2}}$$

$$\boxed{\Delta\beta_{\mathrm{P}} = \begin{cases} \theta_0 - \beta_{\mathrm{P}} \leq \Delta\beta_{\mathrm{P,max}}, \text{foreground} \\ \theta_0' - \beta_{\mathrm{P}} \leq \Delta\beta_{\mathrm{P,max}}, \text{background} \end{cases}}$$



Maximum angular impact parameter

3. Detectable upto a maximum redshift, where the Einstein angle is barely resolved.

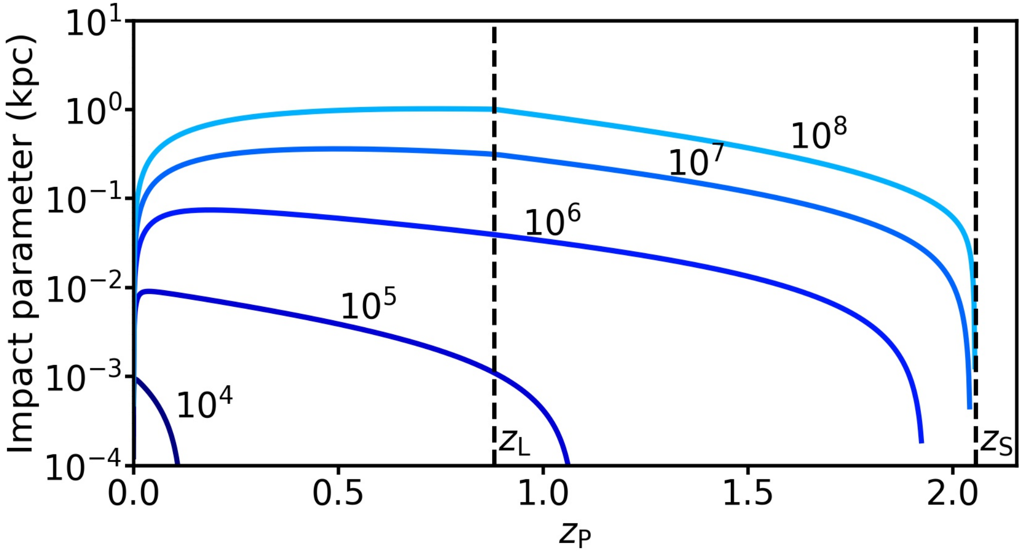
Effective comoving volume

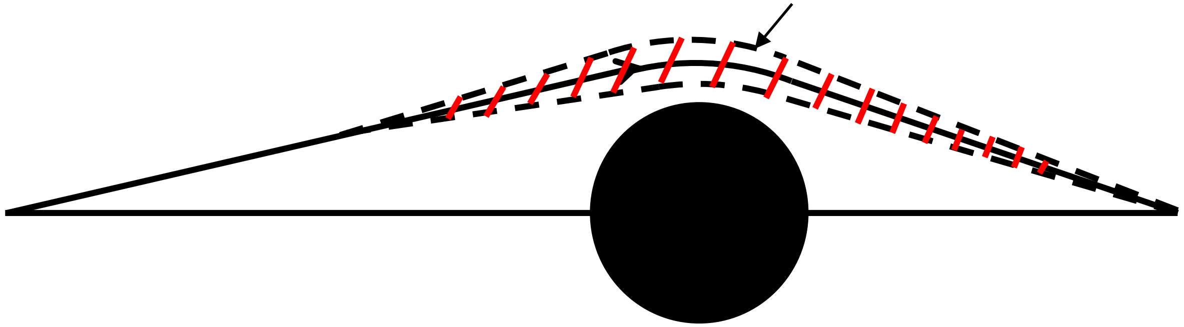
$$\boxed{V_{\mathrm{eff}}(M_{\mathrm{P}}) = \int_{0}^{z_{\mathrm{P,max}}} \Omega(z)\,\frac{d^2V}{d\Omega dz}\,dz}$$

$$\boxed{\Omega(z) \approx 2\omega_{\mathrm{arc}}\,\Delta\beta_{\mathrm{P,max}}}$$

Maximum angular impact parameter

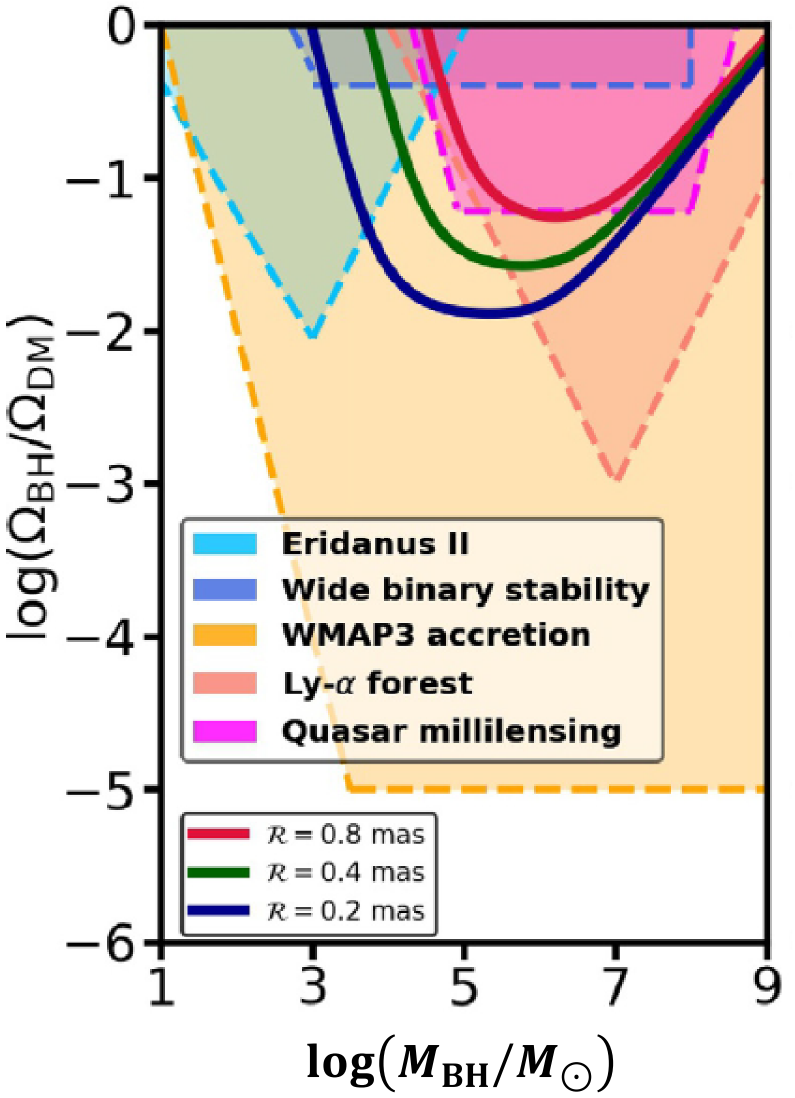
Arc-length





Volume of detectability

Constraints on mass density of BHs



Null detection → **Upper bound**

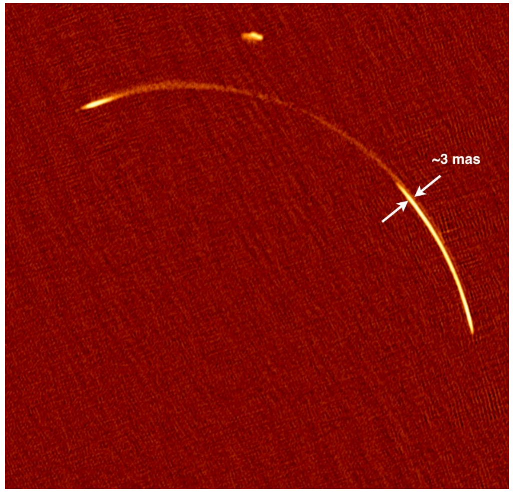
95% confidence level

Low mass

$$\frac{\Omega_{BH}}{\Omega_{DM}} \sim \frac{\mathcal{R}^5}{M_P^2}$$

High mass

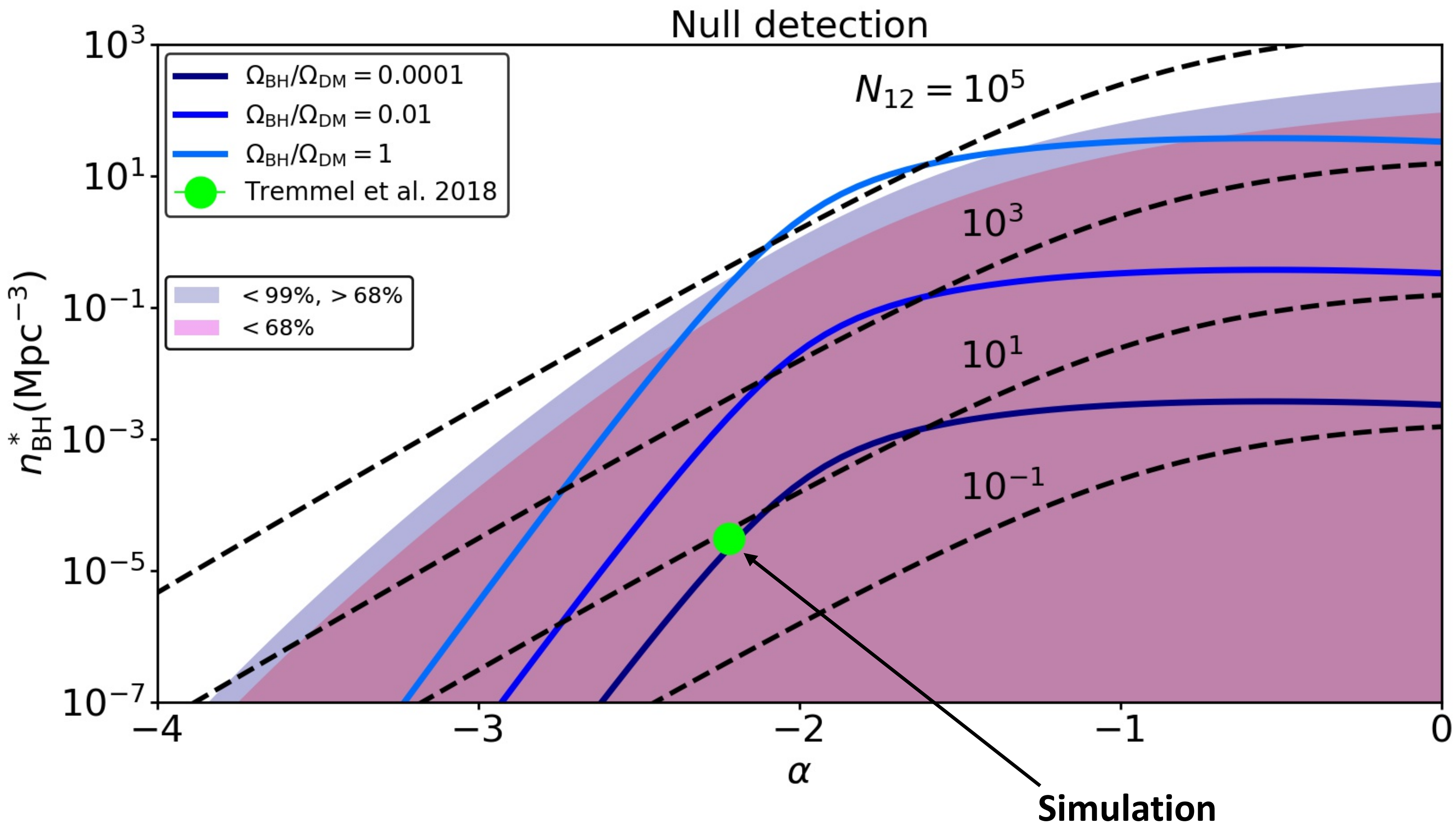
$$\frac{\Omega_{BH}}{\Omega_{DM}} \sim M_P^{\frac{2}{3}} \mathcal{R}^{\frac{1}{3}}$$



John McKean, on behalf of SHARP collaboration

- $10^3 - 10^4$ lenses are required for constraints competitive with WMAP3.
- Probes both PBHs and galactic BHs unlike WMAP3.

Constraints on mass function of BHs



Schechter Mass function of wandering BHs in galactic halos

$$\frac{dn}{dM_{BH}} = \frac{n^*_{BH}}{M^*_{BH}} \left(\frac{M_{BH}}{M^*_{BH}}\right)^{\alpha} e^{-\left(\frac{M_{BH}}{M^*_{BH}}\right)}$$

Average occupation no of SMBHs in halos

$$\langle N_{6+}\rangle(M_{\mathrm{h}}) = N_{12}\left(\frac{M_{\mathrm{h}}}{10^{12} M_{\odot}}\right)$$

$\sim 10^3 - 10^4$ high resolution lenses are required to detect these free-floating BHs.

Summary

- Free-floating black holes- primordial/ galaxy evolution
- Black holes are extremely compact- they cause localized distortion of lensing arcs formed by dark matter halos
- Null detection (single mass species of primordial/ galactic BHs) for one lens configuration puts the strongest constraint of $\frac{\Omega_{BH}}{\Omega_{DM}} \lesssim 10^{-1}$ at $M_{BH} = 10^6 M_\odot$ with 95% confidence at fiducial FWHM of $\mathcal{R} = 0.8$ mas.
- In order to test current predictions for the number of galactic BHs, of order 1000 lenses are required. Future radio surveys with the Square Kilometer Array (SKA) and follow-up by VLBI can achieve this.

Back-up slides

Multiple lens equation

- With perturbing BH

$$\boxed{\overrightarrow{\boldsymbol{\beta}_{\mathrm{S}}} = \vec{\boldsymbol{\theta}} - \vec{\boldsymbol{\alpha}}_{\mathrm{P}}(\vec{\boldsymbol{\theta}}) - \vec{\boldsymbol{\alpha}}_{\mathrm{L}}(\vec{\boldsymbol{\theta}}')}$$

Foreground

$$\vec{\theta}' = \vec{\theta} - \gamma_{\mathrm{f}}\, \vec{\alpha}_{\mathrm{P}}(\vec{\theta}),\ \gamma_{\mathrm{f}} = \frac{D_{\mathrm{PL}}}{D_{\mathrm{L}}} \frac{D_{\mathrm{S}}}{D_{\mathrm{PS}}}$$

$$\boxed{\overrightarrow{\boldsymbol{\beta}_{\mathrm{S}}} = \vec{\boldsymbol{\theta}} - \vec{\boldsymbol{\alpha}}_{\mathrm{L}}(\vec{\boldsymbol{\theta}}) - \vec{\boldsymbol{\alpha}}_{\mathrm{P}}(\vec{\boldsymbol{\theta}}')}$$

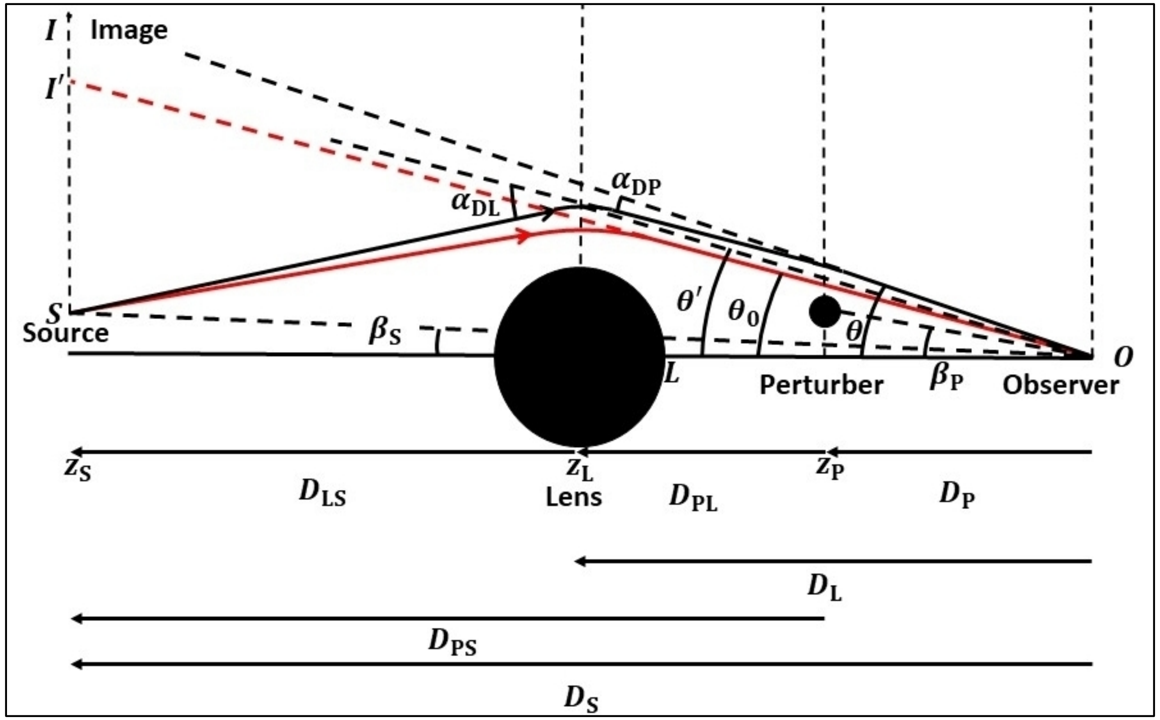
Background

$$\vec{\theta}' = \vec{\theta} - \gamma_{\mathrm{b}}\, \vec{\alpha}_{\mathrm{L}}(\vec{\theta}),\ \gamma_{\mathrm{b}} = \frac{D_{\mathrm{LP}}}{D_{\mathrm{P}}} \frac{D_{\mathrm{S}}}{D_{\mathrm{LS}}}$$

- Without perturbing BH

$$\boxed{\overrightarrow{\boldsymbol{\beta}_{\mathrm{S}}} = \vec{\boldsymbol{\theta}}_0 - \vec{\boldsymbol{\alpha}}_{\mathrm{L}}(\vec{\boldsymbol{\theta}}_0)}$$

Foreground



$$\vec{\alpha}_{\mathrm{P}} = \frac{D_{\mathrm{PS}}}{D_{\mathrm{S}}} \vec{\alpha}_{\mathrm{DP}} \qquad \vec{\alpha}_{\mathrm{L}} = \frac{D_{\mathrm{LS}}}{D_{\mathrm{S}}} \vec{\alpha}_{\mathrm{DL}}$$

Lens models

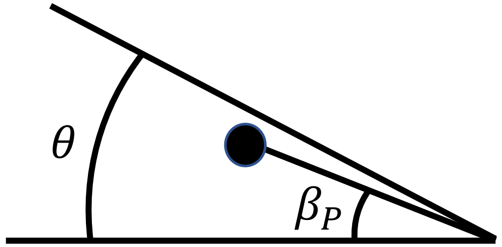
- Primary lens- Dark matter halo (isothermal sphere)

$$\boxed{\vec{\boldsymbol{\alpha}}_{\mathbf{L}} = \boldsymbol{\theta}_{\mathbf{E}} \frac{\vec{\boldsymbol{\theta}}}{|\vec{\boldsymbol{\theta}}|}}$$

Einstein angle $\theta_{\mathrm{E}} = \sqrt{\frac{2\pi G M_{\mathrm{L}}(\theta_{\mathrm{E}})}{c^2} \frac{D_{\mathrm{LS}}}{D_{\mathrm{L}} D_{\mathrm{S}}}}$ ($M_{\mathrm{L}}(\theta_{\mathrm{E}})$ is the 3D mass inside θ_{E}.)

- Perturber- Schwarzschild black hole

$$\boxed{\vec{\boldsymbol{\alpha}}_{\mathbf{P}} = \boldsymbol{\theta}_{\mathbf{EP}}^{2} \frac{\vec{\boldsymbol{\theta}} - \vec{\boldsymbol{\beta}}_{\boldsymbol{P}}}{|\vec{\boldsymbol{\theta}} - \vec{\boldsymbol{\beta}}_{\boldsymbol{P}}|^{2}}}$$



Perturber mass

Einstein angle $\theta_{\mathrm{EP}} = \sqrt{2R_{\mathrm{s}} \frac{D_{\mathrm{PS}}}{D_{\mathrm{P}} D_{\mathrm{S}}}}$ ($R_s = \frac{2GM_P}{c^2}$ is the Schwarzschild radius.)

Perturbative solution

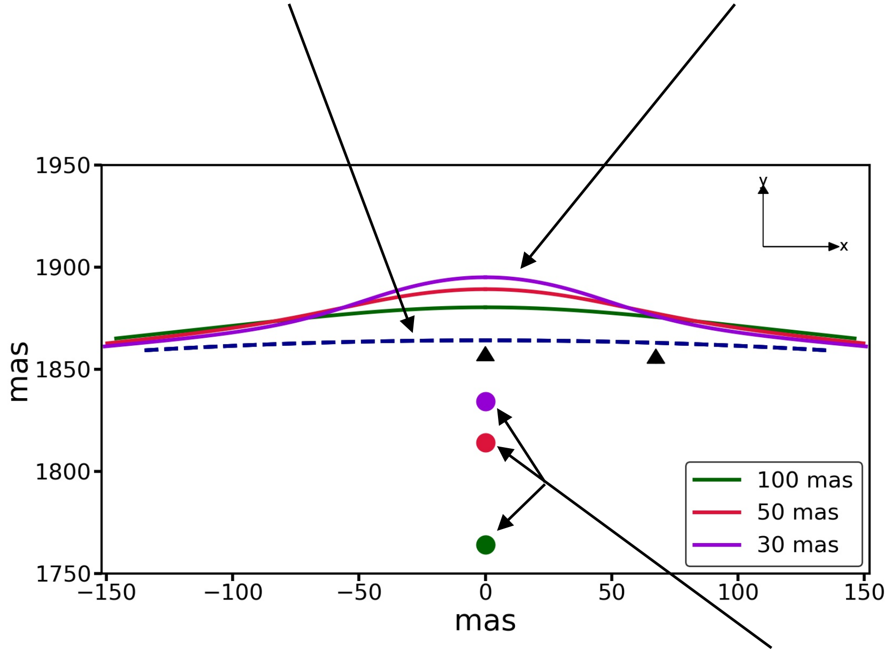
$$\boxed{\delta\vec{\boldsymbol{\theta}} = \vec{\boldsymbol{\theta}} - \vec{\boldsymbol{\theta}}_{\mathbf{0}} = \begin{cases} \mathbb{M}(\vec{\boldsymbol{\theta}}_{\mathbf{0}})\mathbb{C}(\vec{\boldsymbol{\theta}}_{\mathbf{0}})\vec{\boldsymbol{\alpha}}_{\boldsymbol{P}}(\vec{\boldsymbol{\theta}}), \text{foreground} \\ \mathbb{M}(\vec{\boldsymbol{\theta}}_{\mathbf{0}})\vec{\boldsymbol{\alpha}}_{\boldsymbol{P}}(\vec{\boldsymbol{\theta}}'), \text{background} \end{cases}}$$

Magnification tensor $\mathbb{M}(\vec{\theta}_{\mathbf{0}}) = \left(1 - \vec{\nabla}_{\theta}\alpha_{\mathrm{L}}(\theta_0)\right)^{-1}$

Correction tensor $\mathbb{C}(\vec{\theta}_{\mathbf{0}}) = 1 - \gamma_{\mathrm{f}}\vec{\nabla}_{\theta}\alpha_{\mathrm{L}}(\theta_0)$

Unperturbed arc

Perturbed arc



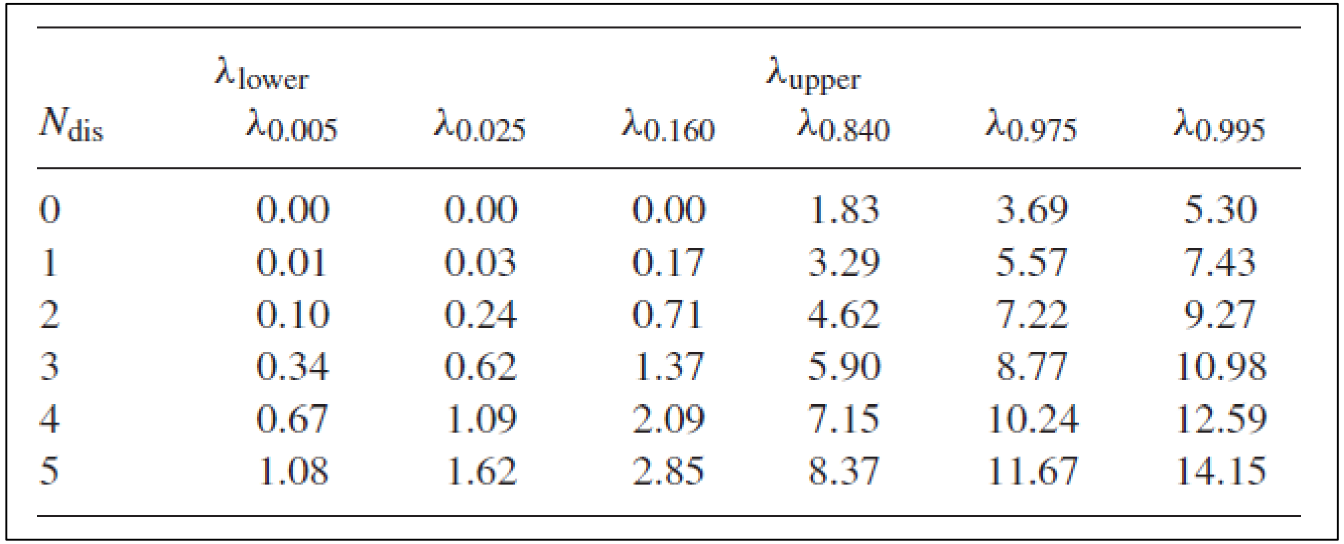
Black hole

Isothermal sphere lens

$$\mathbb{M}(\vec{\theta}_{\mathbf{0}}) = \frac{1}{1 - \frac{\theta_E}{\theta_0}} \begin{bmatrix} 1 - \theta_E \frac{\theta_{0x}^2}{\theta_0^3} & -\theta_E \frac{\theta_{0x}\theta_{0y}}{\theta_0^3} \\ -\theta_E \frac{\theta_{0x}\theta_{0y}}{\theta_0^3} & 1 - \theta_E \frac{\theta_{0y}^2}{\theta_0^3} \end{bmatrix} \qquad \mathbb{C}(\vec{\theta}_{\mathbf{0}}) = \begin{bmatrix} 1 - \gamma_{\mathrm{f}}\theta_E \frac{\theta_{0x}^2}{\theta_0^3} & \gamma_{\mathrm{f}}\theta_E \frac{\theta_{0x}\theta_{0y}}{\theta_0^3} \\ \gamma_{\mathrm{f}}\theta_E \frac{\theta_{0x}\theta_{0y}}{\theta_0^3} & 1 - \gamma_{\mathrm{f}}\theta_E \frac{\theta_{0y}^2}{\theta_0^3} \end{bmatrix}$$

Number distribution

- Poisson distribution



	λ_{lower}			λ_{upper}		
N_{dis}	$\lambda_{0.005}$	$\lambda_{0.025}$	$\lambda_{0.160}$	$\lambda_{0.840}$	$\lambda_{0.975}$	$\lambda_{0.995}$
0	0.00	0.00	0.00	1.83	3.69	5.30
1	0.01	0.03	0.17	3.29	5.57	7.43
2	0.10	0.24	0.71	4.62	7.22	9.27
3	0.34	0.62	1.37	5.90	8.77	10.98
4	0.67	1.09	2.09	7.15	10.24	12.59
5	1.08	1.62	2.85	8.37	11.67	14.15

- Comoving number density

No of detections N_{dis}

$$\boxed{\frac{\lambda_{\text{lower}}}{V_{\text{eff}}(M_{\text{BH}})} \leq n_{\text{BH}} \leq \frac{\lambda_{\text{upper}}}{V_{\text{eff}}(M_{\text{BH}})}}$$

Constraints on ρ_{BH}

Constraints on mass function of wandering BHs

- Formed as a result of galaxy evolution
- Follow **Schechter mass function**-

$$\frac{dn}{dM_{BH}} = \frac{n^*_{BH}}{M^*_{BH}} \left(\frac{M_{BH}}{M^*_{BH}}\right)^{\alpha} e^{-\left(\frac{M_{BH}}{M^*_{BH}}\right)}$$

Mass cut-off

$\sim 10^9 M_\odot$

Mass density

$$\rho_{BH} = \int_{M_{min}}^{\infty} M_{BH} \frac{dn}{dM_{BH}} dM_{BH}$$
$$= n^*_{BH} M^*_{BH} \Gamma(\alpha + 2, M_{min}/M^*_{BH})$$

Null detection

$$\lambda_{dis} \leq \begin{cases} 1.83, 68\% \\ 3.69, 95\% \\ 5.30, 99\% \end{cases}$$

Constraints on the expected no

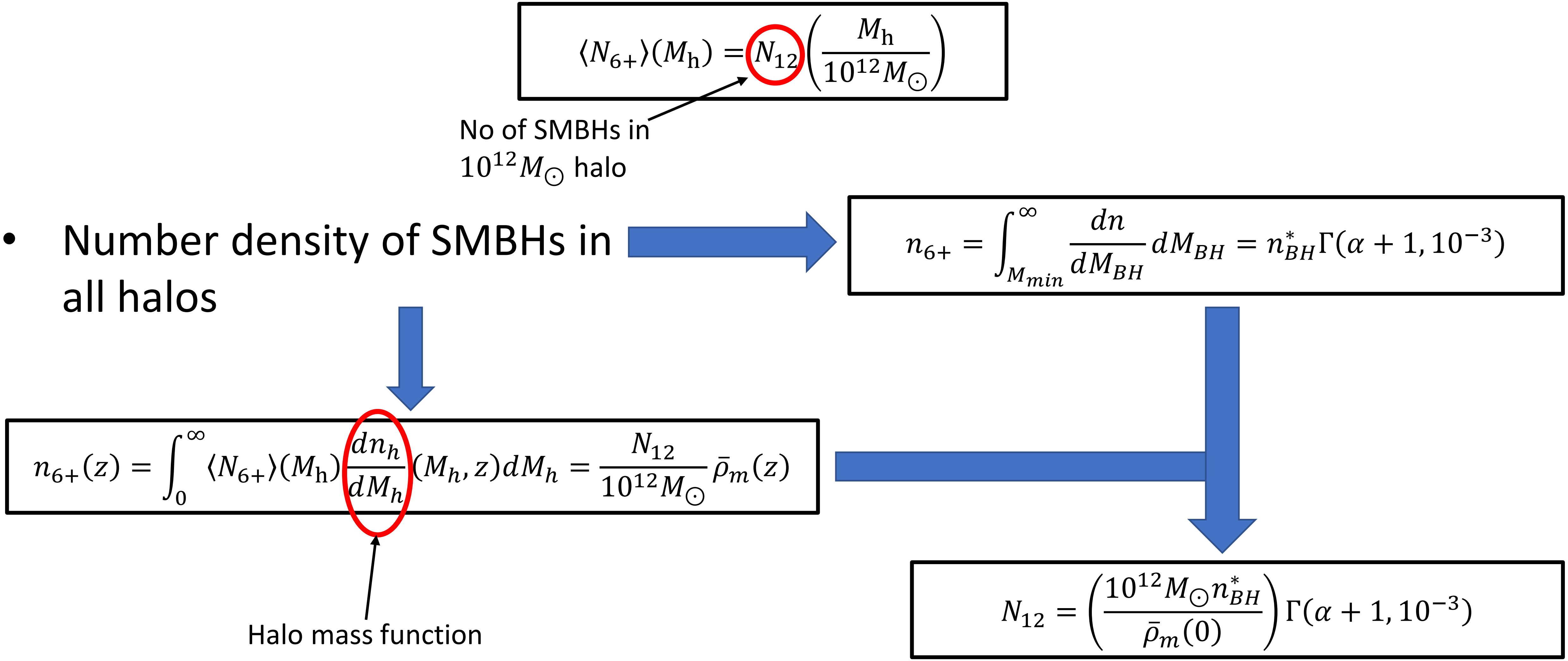
$$\lambda_{dis} = \int_0^{\infty} V_{eff}(M_{BH}) \frac{dn}{dM_{BH}} dM_{BH}$$

from detection/not

Constraints on n^*_{BH} and α

Occupation number of wandering SMBHs

- Average number of wandering SMBHs $\left(M_{\rm BH} \geq 10^6 M_\odot\right)$ in a halo of mass $M_{\rm h}$



$$\langle N_{6+}\rangle(M_{\rm h}) = N_{12}\left(\frac{M_{\rm h}}{10^{12}M_\odot}\right)$$

No of SMBHs in $10^{12}M_\odot$ halo

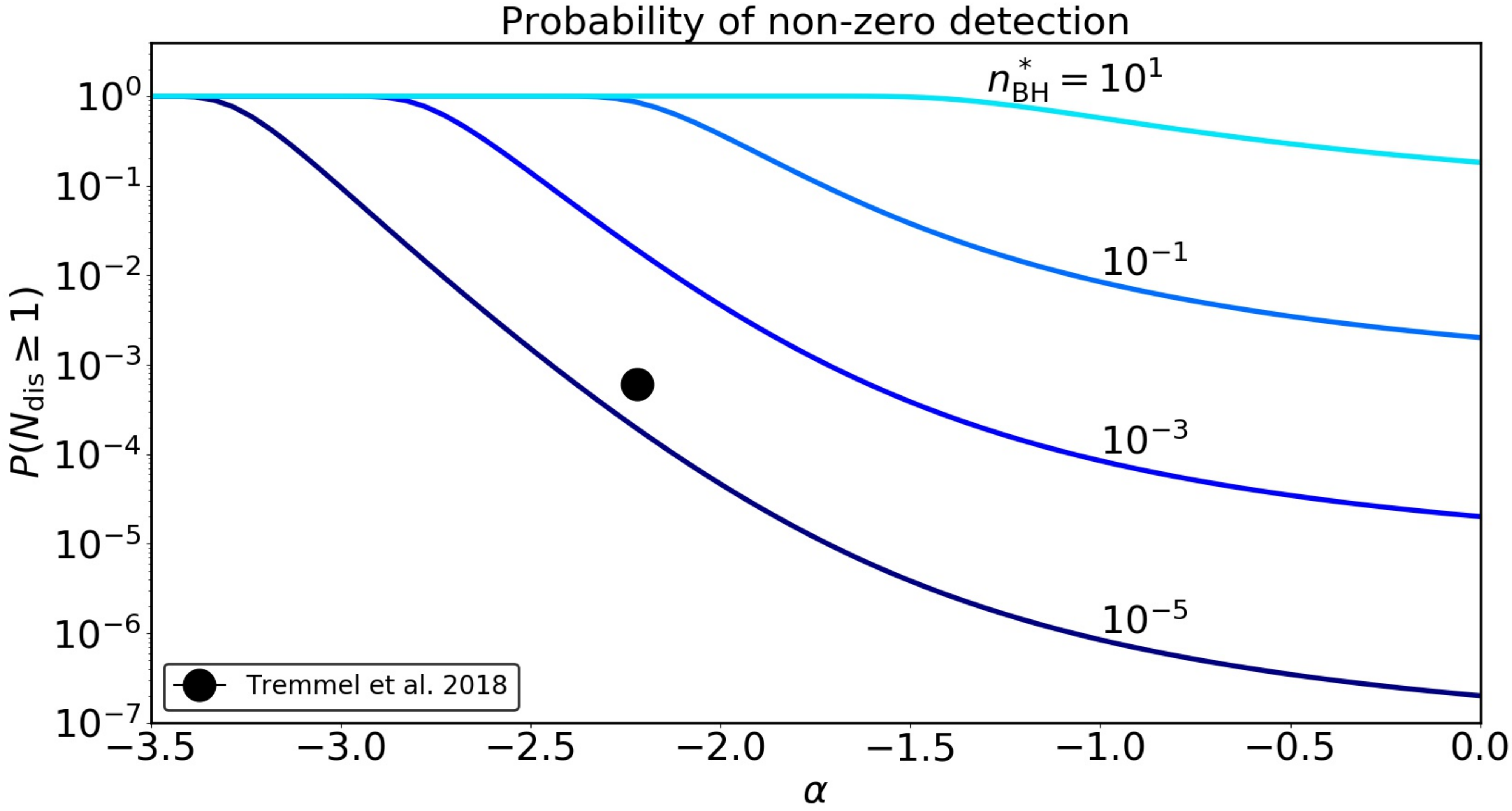
- Number density of SMBHs in all halos

$$n_{6+} = \int_{M_{min}}^{\infty} \frac{dn}{dM_{BH}} dM_{BH} = n_{BH}^{*}\Gamma(\alpha+1, 10^{-3})$$

$$n_{6+}(z) = \int_0^{\infty} \langle N_{6+}\rangle(M_{\rm h}) \frac{dn_h}{dM_h}(M_h, z) dM_h = \frac{N_{12}}{10^{12}M_\odot}\bar{\rho}_m(z)$$

Halo mass function

$$N_{12} = \left(\frac{10^{12}M_\odot n_{BH}^{*}}{\bar{\rho}_m(0)}\right)\Gamma(\alpha+1, 10^{-3})$$



$$\boldsymbol{P(N_{\mathrm{dis}} \geq 1) = 1 - e^{-\lambda_{\mathrm{dis}}}}$$

$\sim 10^3$ high resolution lenses are required for detection.