Radiative Transfer Modelling

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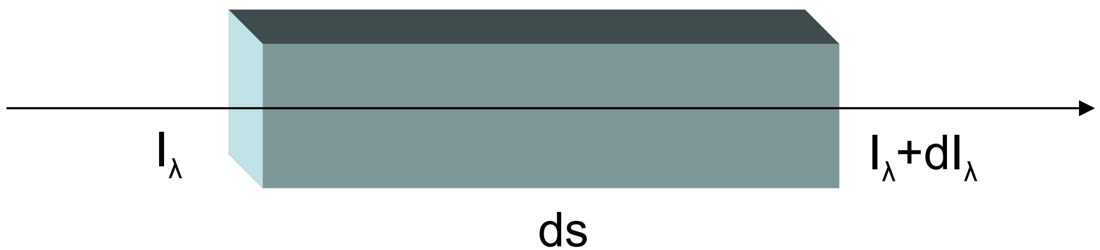
Absorption coefficient and optical depth

• Consider radiation shining through a layer of material. The intensity of light is found experimentally to decrease by an amount dI_λ where $dI_\lambda = -\alpha_\lambda I_\lambda ds$. Here, ds is a length and α_λ is the absorption coefficient [cm^{-1}]. The photon mean free path, l, is inversely proportional to α_λ.



• Two physical processes contribute to light attenuation: (i) absorption where the photons are destroyed and the energy gets thermalized and (ii) scattering where the photon is shifted in direction and removed from the solid angle under consideration.

• The radiation sees a combination of α_λ and ds over some path length L, given by a dimensional quantity, the optical depth:

$$\tau_\lambda = \int_0^L a_\lambda ds$$

Importance of optical depth

We can write the change in intensity over a path length as $dI_\lambda = -I_\lambda d\tau_\lambda$.
This can be directly integrated and give the extinction law:

$$I_\lambda(\tau_\lambda) = I_\lambda(0)e^{-\tau_\lambda}$$

- An optical depth of $\tau_\lambda = 0$ corresponds to no reduction in intensity.
- An optical depth of $\tau_\lambda = 1$ corresponds to a reduction in intensity by a factor of e=2.7.

This defines the "optically thin" – "optically thick" limit.

- For large optical depths $\tau_\lambda >> 1$ negligible intensity reaches the observer.

Emission coefficient and source function

• We can also treat emission processes in the same way as absorption by defining an emission coefficient, ε_λ [erg/s/cm^3/str/cm] :

$$dI_\lambda = \varepsilon_\lambda ds$$

• Physical processes that contribute to ε_λ are (i) real emission – the creation of photons and (ii) scattering of photons to the direction being considered.

• The ratio of emission to absorption is called the source function.

$$S_\lambda = \varepsilon_\lambda / \alpha_\lambda$$

Radiative Transfer Equation

- We can now incorporate the effects of emission and absorption into a single equation giving the variation of the intensity along the line of sight. The combined expression is:

$$dI_\lambda = -\alpha_\lambda I_\lambda ds + \varepsilon_\lambda ds$$

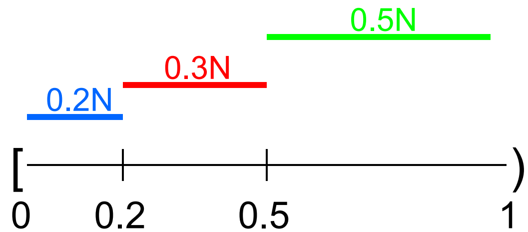
or, in terms of the optical depth and the source function the equation becomes:

$$\frac{dI_\lambda}{d\tau_\lambda} = -I_\lambda + S_\lambda$$

- Once α_λ and ε_λ are known it is relatively easy to solve the radiative transfer equation. When scattering though is present, solution of the radiative transfer equation is more difficult.

Monte Carlo – The method

- Process with 3 outcomes: Outcome A with probability 0.2
  Outcome B with probability 0.3
  Outcome C with probability 0.5



We select a **large** number N of **random numbers r** uniformly distributed in the interval [0, 1). Approximately, 0.2N will fall in the interval [0, 0.2), 0.3N in the interval [0.2, 0.5) and 0.5N in the interval [0.5, 1). **The value of a random number r uniquely determines one of the three outcomes**.

- More generally:

If $E_1, E_2, \ldots, E_n$ are n independent events with probabilities $p_1, p_2, \ldots, p_n$ and $p_1 + p_2 + \ldots + p_n = 1$ then a random number r with

$$p_1+p_2+\ldots+p_{i-1} \leq r < p_1+p_2+\ldots+p_i$$

determines event E_i

- For continuous distributions:

If $p(x)dx$ is the probability for event $x(a \leq x < b)$ to occur then

$$\int_a^x p(\xi)d\xi = r$$

determines event x uniquely.

Monte Carlo – Procedure

Step 1: Consider a photon that was emitted at position (x_0, y_0, z_0).

Step 2: To select a random direction (θ, φ), pick a random number r and set φ=2πr and another random number r and set cosθ=2r-1.

Step 3: To find a step s that a photon makes before an event (scattering or absorption) occurs, pick a random number r and use the probability density

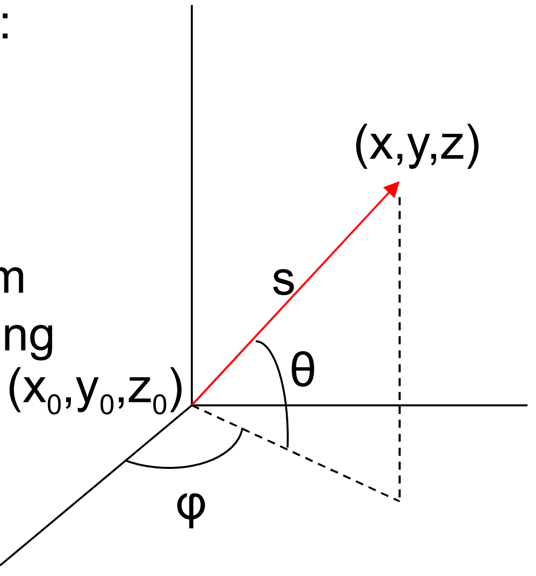
$$p(\xi)=(1/l)e^{-\xi/l}$$ where l is the mean free path of the medium

in equation $$\int_a^s p(\xi)d\xi = r$$

Step 4: The position of the event in space is at (x, y, z), where:

$$x= x_0+ s \sin\theta \cos\varphi$$
$$y= y_0+ s \sin\theta \sin\varphi$$
$$z= z_0+ s \cos\theta$$

Step 5: To determine the kind of event occurred, pick a random number r. If r<ω (the albedo) the event was a scattering (go to Step 6), else, it was an absorption (record the energy absorbed and go to Step1).



Monte Carlo – Procedure

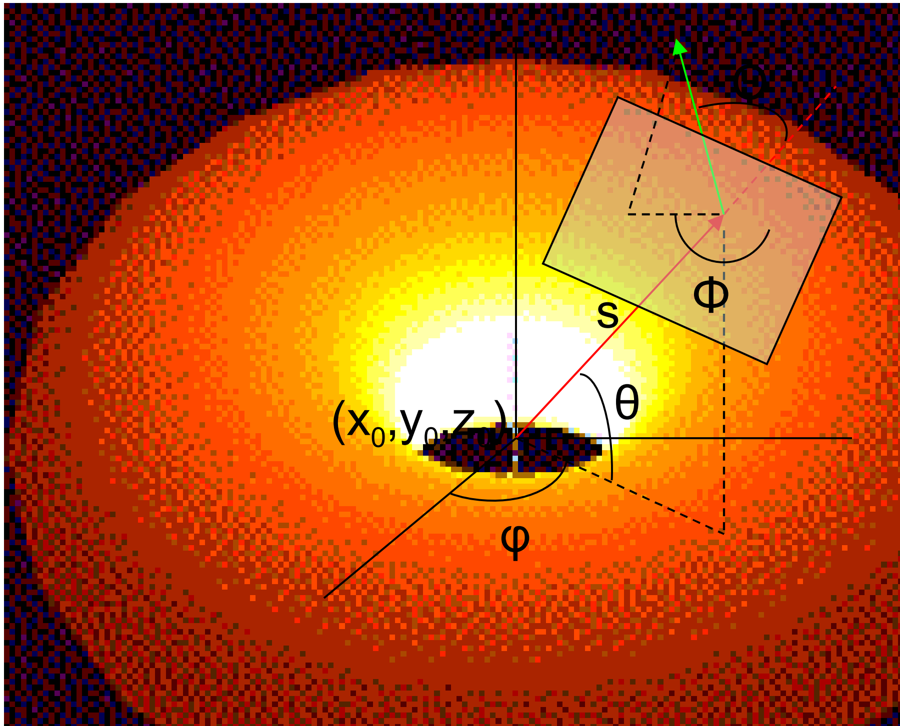
Step 6: Determine the new direction (Θ, Φ), where Θ is the angle between the old and the new direction (to be determined from the Henyey-Greenstein phase function *) and Φ=2πr.

Step 7: Convert (Θ, Φ) into (θ, φ) and go to Step 3.

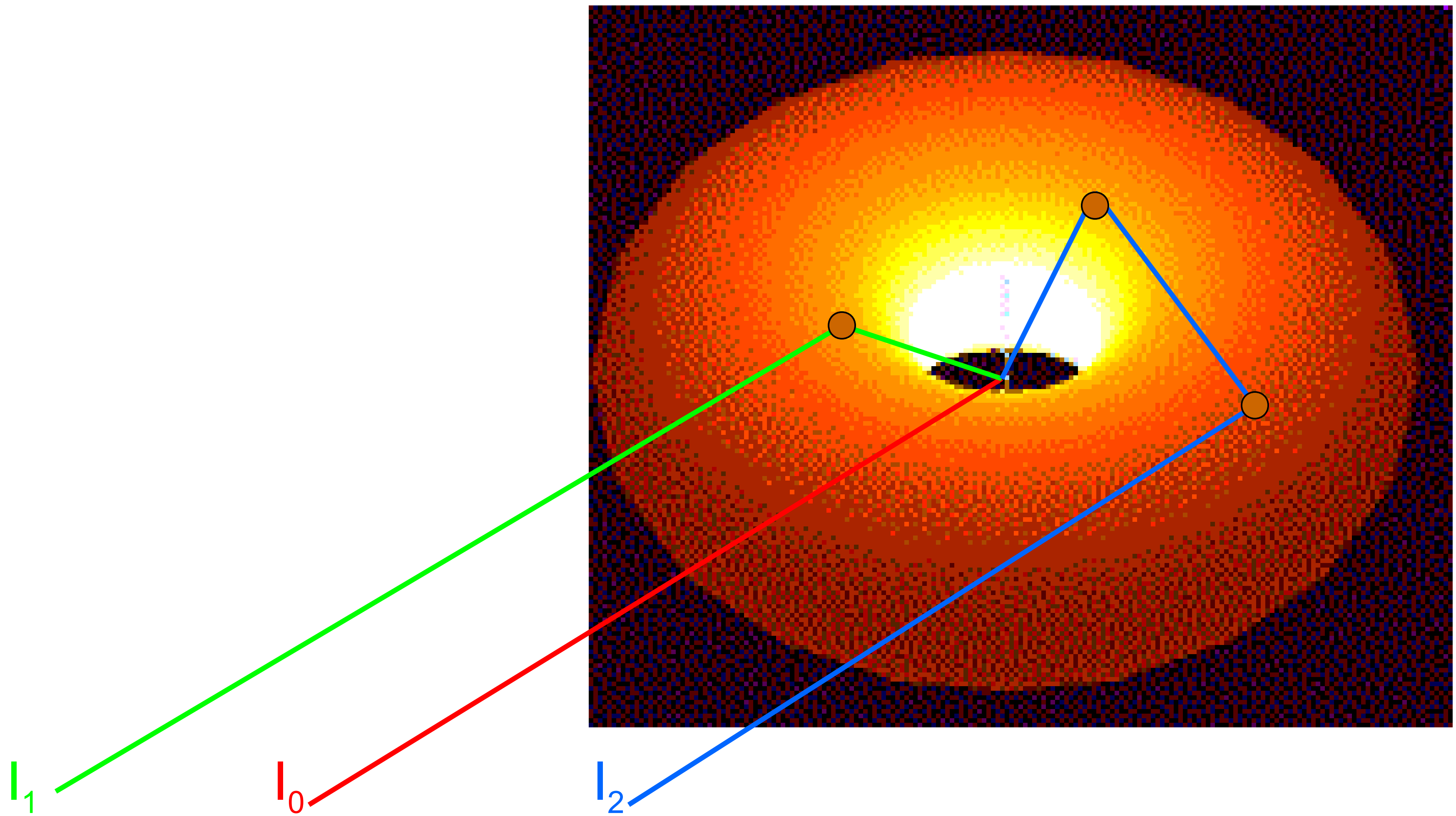
Continue the loop described above until the photon is either absorbed or escapes from the absorbing medium.

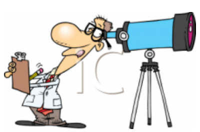
$$^{*}p(\hat{n}',\hat{n}) = \frac{1-g^2}{(1+g^2-2g\hat{n}'\cdot\hat{n})^{3/2}}$$

Henyey & Greenstein, 1941, ApJ, **93**, 70



Scattered intensities - The method





$$I = I_0 + I_1 + I_2 + \ldots$$

Scattered intensities
The method

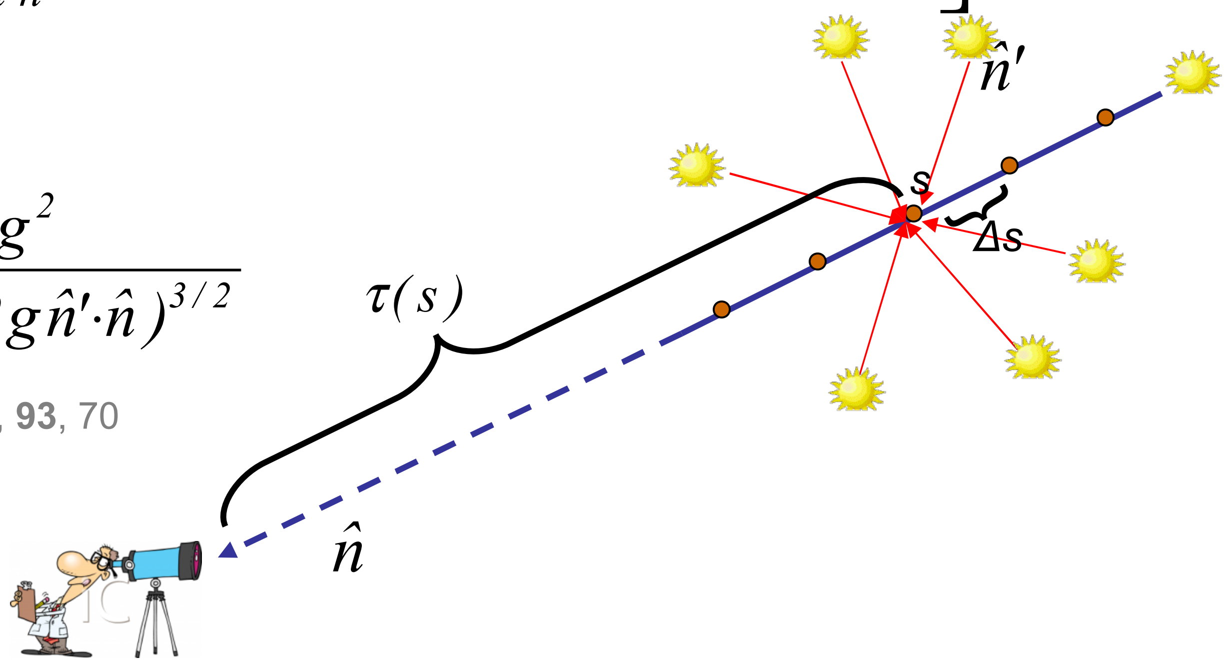
$$I = I_0 + I_1 + I_2 + \ldots.$$

$$I_0 = \sum_{all\,\Delta s} \eta(s)\,\Delta s\, e^{-\tau(s)}$$

$$I_1 = \omega \sum_{all\,\Delta s} \left[\kappa(s) \sum_{all\,\hat{n}} I_0(\hat{n}')\, p(\hat{n}',\hat{n})\,(\Delta\Omega/4\pi)\,\Delta s \right] e^{-\tau(s)}$$

$$p(\hat{n}',\hat{n}) = \frac{1-g^2}{(1+g^2-2g\,\hat{n}'\cdot\hat{n})^{3/2}}$$

Henyey & Greenstein, 1941, ApJ, **93**, 70



$$\omega_V \approx 0.6$$

$$g_V \approx 0.5$$

Weingartner & Draine, 2001, ApJ, **548**, 296

Scattered Intensities - Approximation

$$I = I_0 + I_1 + I_2 + \ldots = I_0\left(1 + \frac{I_1}{I_0} + \frac{I_2}{I_0} + \ldots\right) = I_0\left(1 + \frac{I_1}{I_0} + \frac{I_1}{I_0}\frac{I_2}{I_1} + \ldots\right) \xrightarrow[n \geq 2]{\frac{I_n}{I_{n-1}} \approx \frac{I_1}{I_0}} I_0\left(1 + \frac{I_1}{I_0} + \left(\frac{I_1}{I_0}\right)^2 + \ldots\right) =$$

$$I_0\left(\frac{1}{1 - I_1 / I_0}\right)$$

Kylafis & Bahcall, 1987, ApJ, **317**, 637

Verification

1. The scattering is essentially forward

$$p(\hat{n}', \hat{n}) = \frac{1 - g^2}{(1 + g^2 - 2g\hat{n}' \cdot \hat{n})^{3/2}}$$

Henyey & Greenstein, 1941, ApJ, **93**, 70

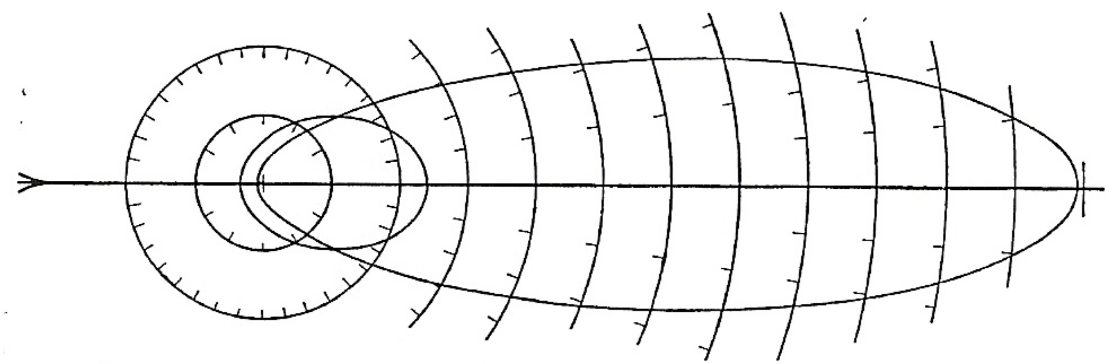


FIG. 3.—Polar diagram of the phase function of equation (2), for $\gamma = 1$. The more elongated curve is for $g = +\frac{2}{3}$; the other, for $g = +\frac{1}{3}$. The radiation is incident on the particle from the left, as shown by the arrow.

Approximation

$$\frac{I_n}{I_{n-1}} \approx \frac{I_1}{I_0}$$

$$I = I_0 + I_1 + I_2 + \ldots = I_0\left(1 + \frac{I_1}{I_0} + \frac{I_2}{I_0} + \ldots\right) = I_0\left(1 + \frac{I_1}{I_0} + \frac{I_1}{I_0}\frac{I_2}{I_1} + \ldots\right) \underset{n \geq 2}{=\!=\!=} I_0\left(1 + \frac{I_1}{I_0} + \left(\frac{I_1}{I_0}\right)^2 + \ldots\right) =$$

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Verification

2. Computation of the I_2 term

$$\tau^e(V) = 100$$

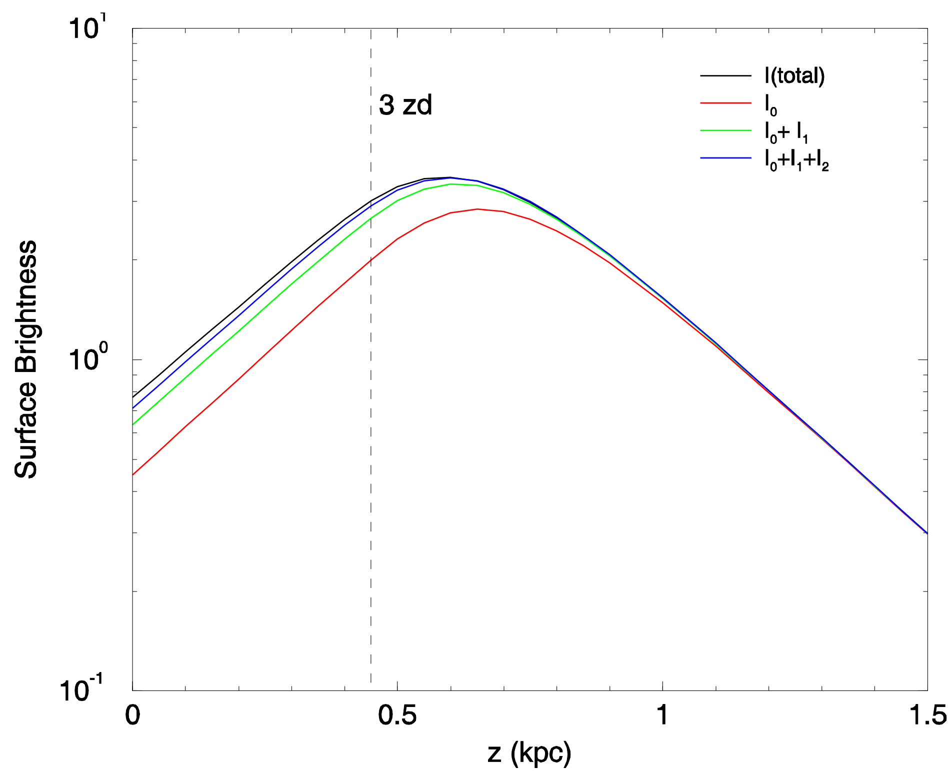
$$\theta = 90^e$$

$$h_s = 3kpc$$

$$h_d = 3kpc$$

$$z_s = 0.3kpc$$

$$z_d = 0.15kpc$$



Approximation

$$I = I_0 + I_1 + I_2 + \ldots = I_0\left(1 + \frac{I_1}{I_0} + \frac{I_2}{I_0} + \ldots\right) = I_0\left(1 + \frac{I_1}{I_0} + \frac{I_1}{I_0}\frac{I_2}{I_1} + \ldots\right) \underset{n \geq 2}{\overset{\frac{I_n}{I_{n-1}} \approx \frac{I_1}{I_0}}{=\!=\!=}} I_0\left(1 + \frac{I_1}{I_0} + \left(\frac{I_1}{I_0}\right)^2 + \ldots\right) = I_0\left(\frac{1}{1 - I_1/I_0}\right)$$

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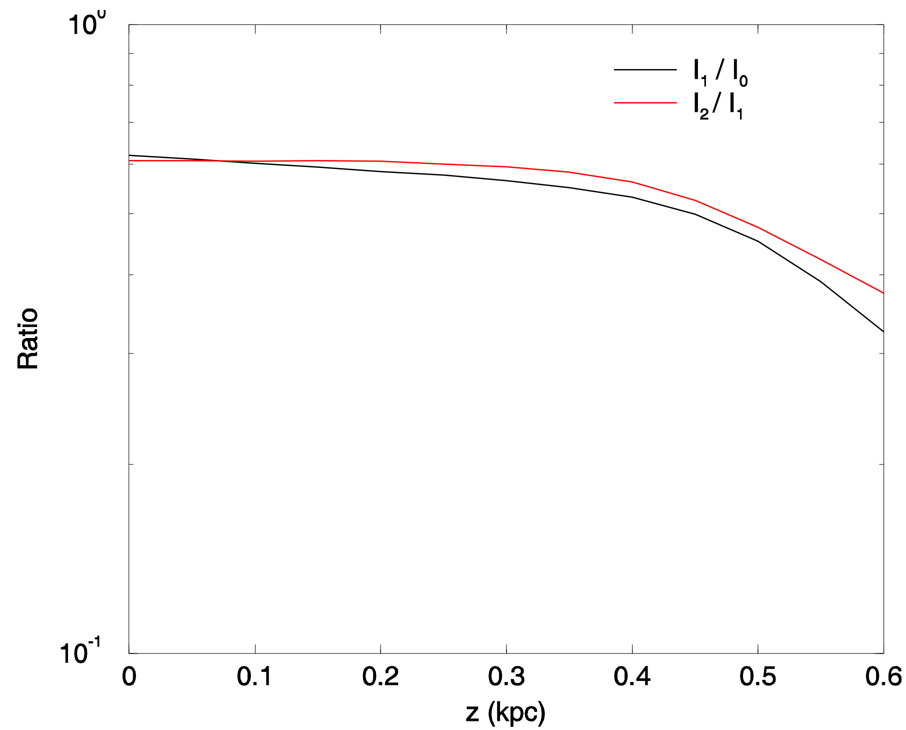

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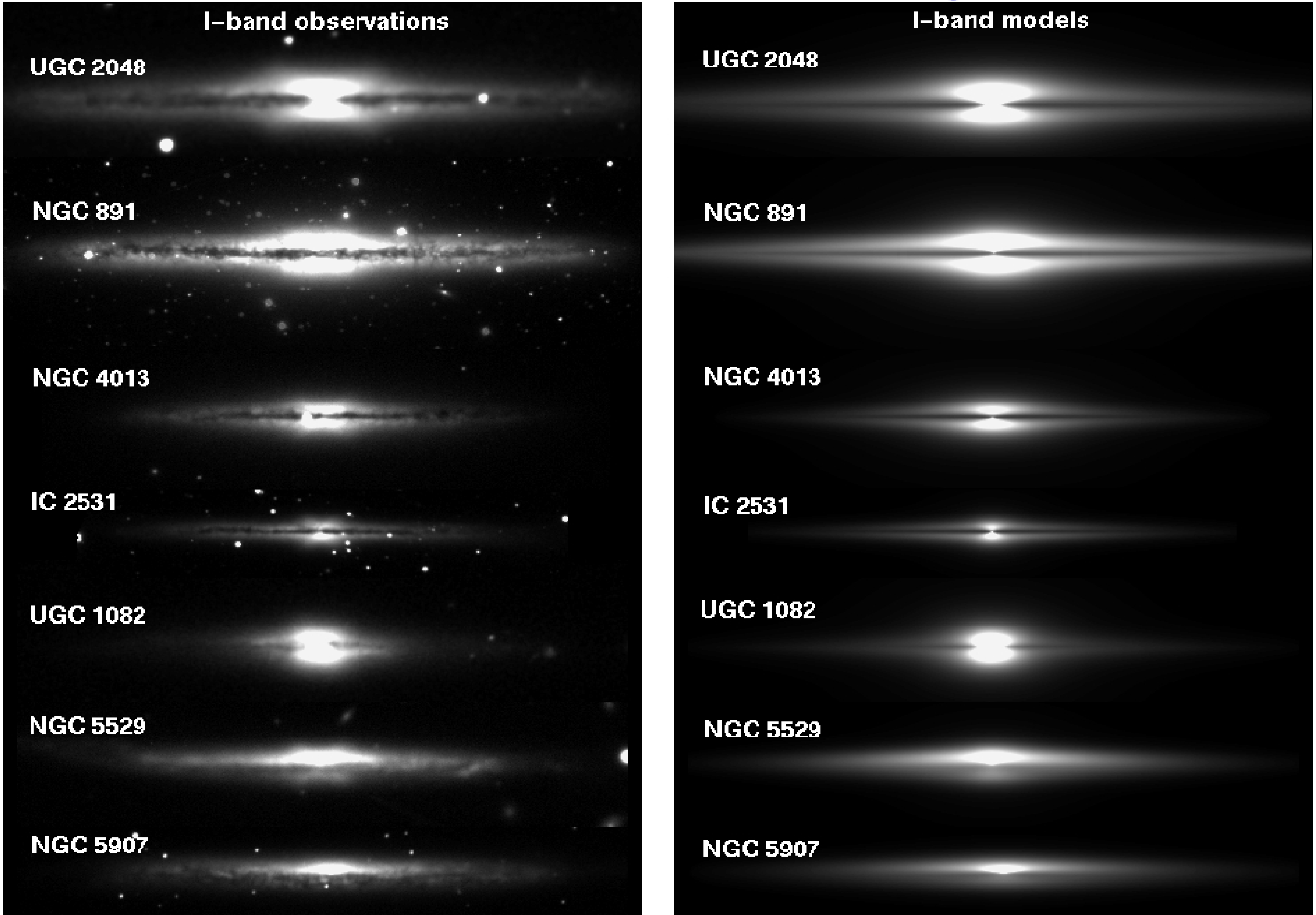
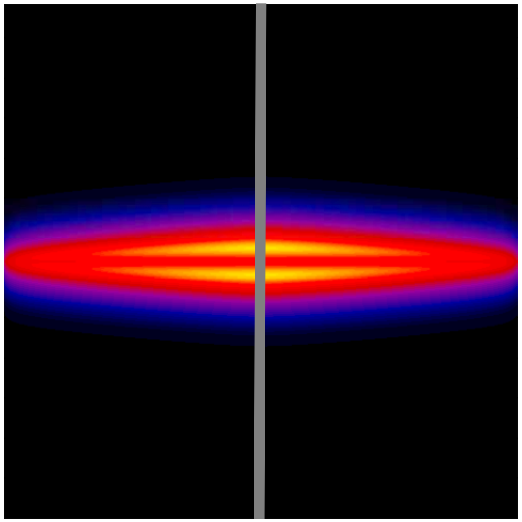
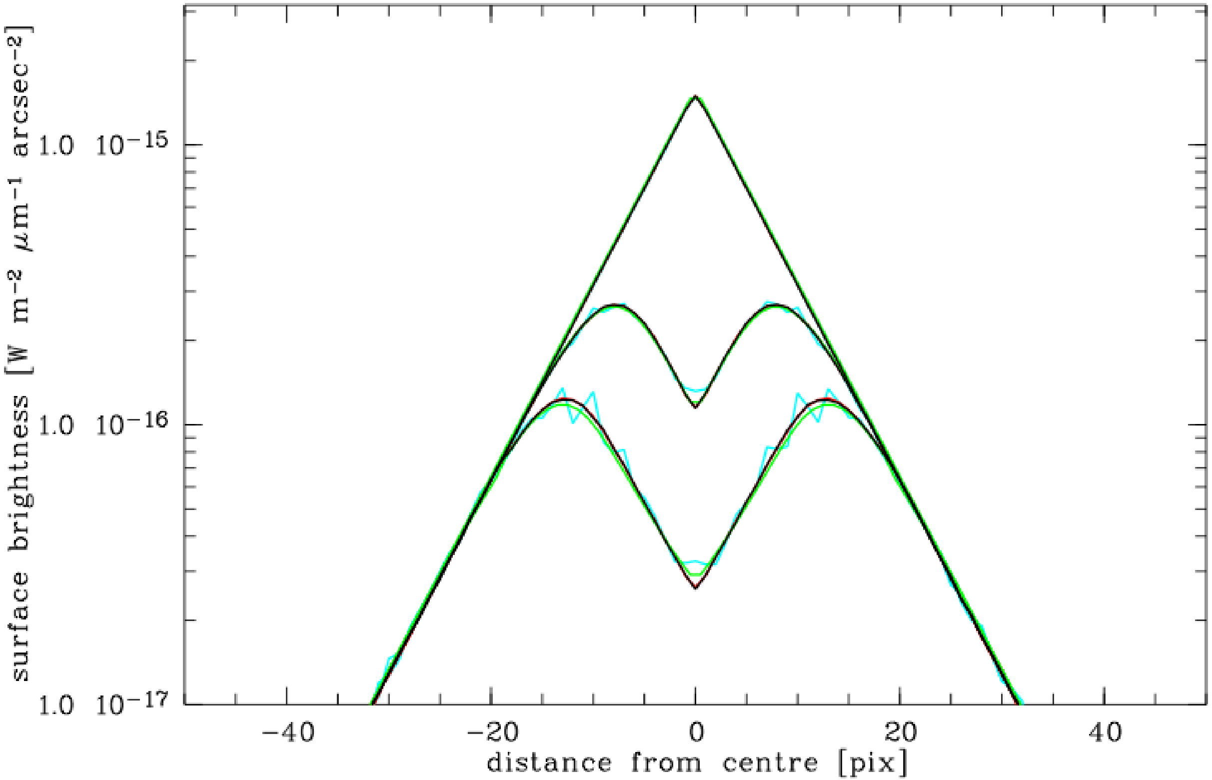
Model application in spiral galaxies



Xilouris et al, 1999, A&A, 344, 868

Code Comparison in galactic environments





DIRTY – M.C.

SKIRT – M.C.

TRADING –M.C.

CRETE – S.I.

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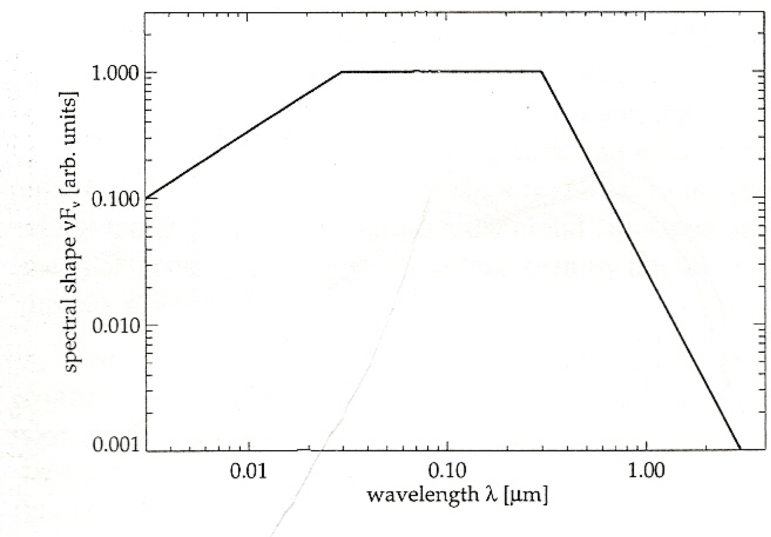
SED of an accretion disk



Models with different inclination angle

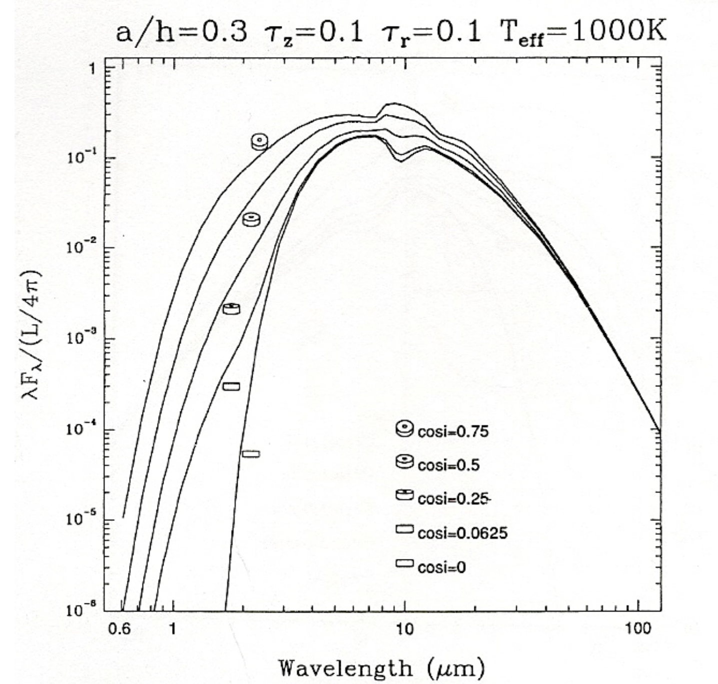
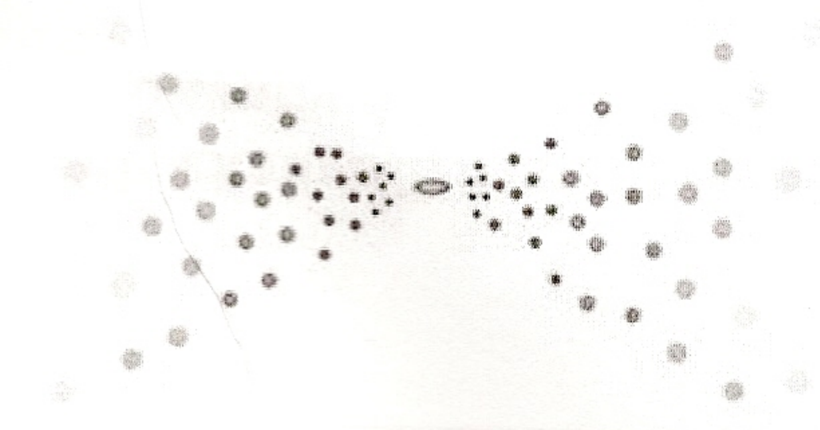


FIG. 5a

Clumpy geometry of a torus



Modeling the SED of NGC 1068